Recursive FFT Implementation for Epilepsy EEG Detection

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***Abstract*—Epilepsy is conventionally monitored via Electroencephalography (EEG), but its a struggle for traditional time-domain analysis to distinguish normal brainwaves from ictal (seizure) states. This paper presents a complete implementation of a Recursive Fast Fourier Transform (FFT) utilizing the Divide and Conquer paradigm to detect epilepsy with frequency domain analysis. Applied to the Bonn EEG dataset, the algorithm successfully extracted relative band powers to classify neurological states. Results demonstrated a dramatic pathological shift during seizures: Theta band power violently increased from 16.05% to 54.01%, while normal Alpha rhythms were suppressed. Using a 50.0% low-frequency ($\Theta + \Delta$) threshold, the system accurately classified the ictal signal at 80.78% low-frequency dominance. Finally, empirical benchmarking validated the $O(N \log N)$ theoretical complexity, demonstrating a $5\times$ execution speedup over the Naïve DFT at $N = 1024$, to ensure scalability for real-time diagnostics.**

***Keywords*—Electroencephalography (EEG), Epilepsy Detection, Fast Fourier Transform (FFT), Divide and Conquer, Frequency Band Analysis**

I. Introduction

Epilepsy is a neurological disorder characterized by hypersynchronous neuronal discharges. While Electroencephalography (EEG) is the primary diagnostic tool, identifying seizures only through time-domain waveforms is challenging due to overlapping frequency rhythms. Raw EEG signals are highly complex composites of multiple overlapping frequency rhythms. While an active seizure often registers at a higher electrical amplitude, the specific pathological shifts in the brain's underlying frequency behavior remain entangled and largely indistinguishable to human observation when plotted linearly against time. In order to overcome this visual ambiguity, signals must be transformed into the frequency domain. However, the standard Discrete Fourier Transform (DFT) possesses a quadratic $O(N^2)$ computational complexity, making real-time processing of high-resolution EEG samples highly prohibitive. By applying a recursive Divide and Conquer algorithm, the Fast Fourier Transform (FFT) reduces this time complexity to $O(N \log N)$, which translates an intractable mathematical operation into a scalable clinical tool.

This study validates the efficiency of the Divide and Conquer FFT and its practical efficacy in medical diagnostics. By utilizing a custom Python pipeline, the research analyzes real-world data from the benchmark Bonn EEG dataset to differentiate normal activity from active ictal states. The objective is to prove that the recursive FFT not only scales efficiently according to the Master Theorem but also successfully extracts frequency band features, specifically Theta rhythm dominance, to serve as a reliable and automated classifier for epilepsy detection.

II. Theoretical Frameworks

A. Electroencephalography and Brainwave Frequency Bands

Electroencephalography (EEG) is a non-invasive neurophysiological technique which measures the electrical activity of the brain by using electrodes placed on the scalp. The signal reflects the sum of postsynaptic potentials produced by large populations of neurons firing. EEG has become one of the most widely used tools in clinical neurology and brain-computer research because of its millisecond-level temporal resolution and sensitivity to neural oscillations.

Raw EEG waveform is a composite signal which contains a bunch of overlapping oscillations that happen simultaneously. In order to interpret that signal, it is decomposed into distinct frequency bands, which are characteristic ranges of oscillations measured in Herts (Hz). They are now recognized as five canonical EEG rhythms, with each band associated with specific brain states and cognitive conditions.

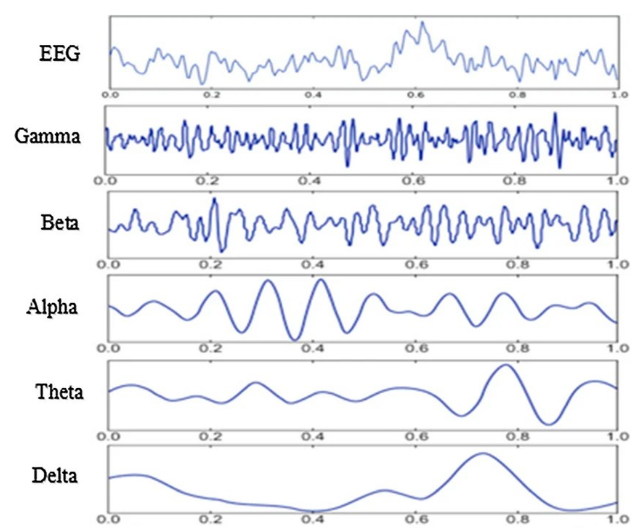

Fig. 1: Visual representations of the five EEG frequency bands [7].

The **Delta band** (0.5-4 Hz) consists of the slowest and highest amplitude oscillations. It is dominant during deep, dreamless sleep, and its presence during alertness is considered pathological.

The **Theta band** (4-8 Hz) is associated with drowsiness, light sleep, deep meditation, and memory consolidation process. Consistently, EEG literature states elevated Theta power in a waking adult signals neurological abnormality and is strongly linked to epilepsy, among other disorders [1].

The **Alpha band** (8-13 Hz) is identified when the brain is idle but in an awake state, this shows Alpha the most easily identifiable rhythm in normal adult EEG.

The **Beta band** (13-30 Hz) is dominant during active cognitive engagement such as focused attention, sensory processing, and motor activity. Abnormally high Beta power is associated with anxiety and certain medicated states, while its decline can indicate pathological slowing [1].

The **Gamma band** (30-100 Hz) represents the fastest EEG oscillations and is associated with high-level cognitive functions. Gamma is the most challenging to measure reliably because of its low amplitude. Its reduction has been associated with depression and schizophrenia [1].

B. The Discrete Fourier Transform

EEG signals represent voltage fluctuations over time. To quantify frequency content, the signal must be transformed into the frequency domain. The mathematical operation that accomplishes this for a digitally sampled signal is the Discrete Fourier Transform (DFT) [4][5].

Given a discrete signal $x[0], x[1], \ldots, x[N-1]$ of length N, the DFT computes N complex-valued output coefficients $X[k]$ according to:

$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j \cdot \frac{2\pi \cdot k \cdot n}{N}}, \quad k = 0, 1, \ldots, N-1 \qquad (1)$$

Each coefficient $X[k]$ captures the amplitude and phase of the sinusoidal component at frequency $f_k = k \cdot \frac{f_s}{N}$, where f_s is the sampling rate in Hz. The magnitude squared of each coefficient, $P[k] = \frac{|X[k]|^2}{N}$, gives the spectral density at that frequency. Plotting $P[k]$ against frequency produces the power spectrum which shows which frequency components carry the most energy in the signal.

The computational limitation of the DFT is that computing all N output coefficients requires N multiplications for each of the N outputs, resulting in a total cost of $O(N^2)$ operations. For a typical EEG segment containing thousands of samples, this quadratic scaling becomes computationally prohibitive, especially when processing hundreds of segments or performing real-time analysis. This limitation motivated the development of the Fast Fourier Transform, which exploits mathematical structure in the DFT to reduce this cost drastically [4].

C. The Fast Fourier Transform as a Divide and Conquer Algorithm

The Fast Fourier Transform (FFT) computes the same result as DFT, but it does it far more efficiently by using symmetry ($W_N^{k+N/2} = -W_N^k$) and periodicity ($W_N^{k+N} = W_N^k$). The most widely used formulation is the Cooley-Tukey radix-2 algorithm, published in 1965 [4], which is a direct application of the Divide and Conquer paradigm.

The key of the algorithm is that a DFT with size N can be expressed as a combination of to DFTs with size $N/2$. One is computed with the even-indexed input samples, and the other one with odd-indexed samples. The decomposition is an algebraic identity that is derived from the periodicity of the exponential. The two half-size DFTs are decomposed recursively in the same way, until the base case of a single-sample input is reached, at which point the DFT is trivially the sample itself [5].

The **divide step** splits the input sequence $x[n]$ of length N into two interleaved half-length sequences containing all even indexes and the other one all odd indexes. The **conquer step** applies the FFT recursively to each half, which produces two sets of $N/2$ frequency coefficients. The **combine step** is known as the butterfly operation which merges two half-results using a complex multiplication by the twiddle factor $W_N^k = e^{-j \cdot 2\pi \cdot k/N}$, and produces the final N-point output. The term "butterfly" refers to the characteristics X-shape formed by the signal flow lines when the combine step is drawn as a circuit diagram.

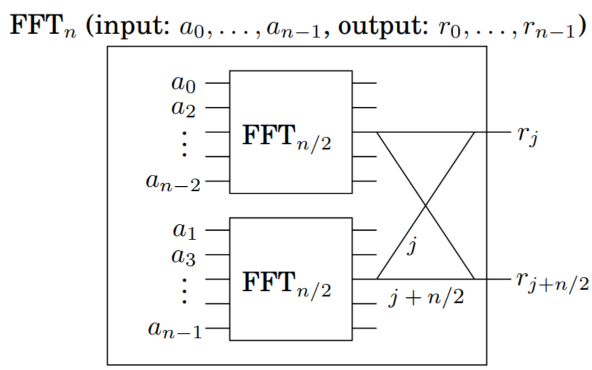

Fig. 2: Divide and conquer structure of the recursive FFT [5].

Even-indexed inputs feed the upper $FFT_{n/2}$ block, odd-indexed inputs feed the lower block. Their outputs are merged by the butterfly operation to produce the final N-point DFT.The efficiency comes from each twiddle factor computation being reused for both the upper and lower halves of the output simultaneously, which cuts the number of multiplications in half at every level of the recursion. For over the $\log_2 N$ levels of recursion, this reuse accumulates into the algorithm's overall complexity.

The computational complexity of the recursive FFT is derived from its recurrence relation. At each level, the problem is divided into two subproblems half the size, and the butterfly combine step requires $O(N)$ work. This gives the recurrence:

$$T(N) = 2 \cdot T(N/2) + O(N) \qquad (2)$$

Applying the **Master Theorem** (Case 2, where $a = 2$, $b = 2$, and $f(N) = O(N) = O(N^{\log_b a})$), this recurrence resolves to:

$$T(N) = O(N \log N) \tag{3}$$

This shows an improvement in scalability compared to DFT. For $N = 4,096$ samples, which is the segment length in the Bonn EEG dataset, the naïve DFT requires approximately 16.7 million operations, whereas the recursive FFT requires only around 49,000. The speed grows with N, which is why FFT is the standard tool for frequency analysis of all but the shortest digital signals [4][5].

D. EEG-Based Epilepsy Detection via Frequency Analysis

Epilepsy is a chronic neurological disorder defined by recurrent, unprovoked seizures caused from abnormal, excessive, or hypersynchronous neuronal discharge in the brain. It is one of the most prevalent neurological conditions worldwide, and EEG analysis is the primary electrophysiological tool used to detect, characterize, and monitor epileptic activity. The characteristic feature of an epileptic seizure in the EEG is a sudden high-amplitude, rhythmic oscillation that disrupts the normal frequency-band distribution of the signal [2].

During an epileptic event or the ictal state, neurons fire in highly synchronized bursts, causing a dramatic spectral shift where power concentrates heavily in lower-frequency bands (like Theta and Delta) that are normally suppressed in alert adults [1][2]. Because normal and epileptic EEGs can look similar in the time domain, Fast Fourier Transform (FFT) is essential to convert the signal into a frequency-domain power spectrum, making this pathological frequency shift directly visible and quantifiable [2].

In order to validate this approach, Nie et al. [2] demonstrated that FFT-based feature extraction successfully isolates epileptic EEG by capturing these band-power differences without requiring complex wavelet decomposition. Following this methodology, the present study uses recursive FFT to compute relative band power as a classification criteria, which focuses heavily on Theta band elevation to distinguish normal (subset A) from ictal (subset E) signals within the Bonn EEG dataset.

E. Band Power Extraction from the Power Spectrum

Once the FFT has been applied to an EEG segment and the power spectrum $P[k] = \frac{|X[k]|^2}{N}$ has been obtained, the remaining step is to quantify how much signal energy falls within each of the five frequency bands. This process is called band power extraction, and it is what converts the raw FFT output into a clinically interpretable set of features.

Each frequency bin k in the FFT output corresponds to a physical frequency $f_k = k \cdot \frac{f_s}{N}$. The band power for a given frequency range $[f_{\text{low}}, f_{\text{high}}]$ is computed by summing the power spectral values of all bins whose corresponding physical frequency falls within that range. The result is a single scalar representing the total energy the signal carries in that frequency region.

By presenting each band's absolute power as a fraction of the total spectral power, raw band power is standardized into relative band power to account for differences in individual anatomy, recording sessions, and electrode placement. The dominant band, the one with the highest relative power, can be used as the main classification feature since this normalization guarantees that the analysis reflects the real distribution of brain-state energy rather than artifactual recording conditions.

Consequently, while a healthy, alert adult (Bonn subset A) is expected to exhibit Alpha dominance, an individual in an ictal state (Bonn subset E) will show a distinct shift toward low-frequency Theta or Delta dominance due to epileptic hypersynchrony [1][2]. The primary method for identifying epilepsy in this study is the distinct division between dominant-band characteristics.

III. Implementation

To practically evaluate the theoretical concepts, a complete data pipeline was implemented in Python. The system architecture is divided into three core functional modules: data preprocessing (`data/loader.py`), recursive frequency transformation (`core/fft.py`), and classification heuristics (`core/classifier.py`).

A. Dataset Acquisition and Preprocessing Pipeline

The algorithm was validated using the Bonn EEG dataset [3]. The evaluation targets two contrasting subsets: **Subset A**, which contains scalp EEG recordings of healthy awake subjects with eyes open (file prefix `Z`), and **Subset E**, containing intracranial recordings of epileptic patients during an active seizure (file prefix `S`). Each segment is 23.6 seconds long, digitized at $f_s = 173.6$ Hz, which yields exactly 4,097 discrete samples per file. The amplitude range difference is stark; Subset A spans approximately $[-254, +169]$ µV, while Subset E spans $[-1{,}026, +727]$ µV, a 4× larger amplitude consistent with intracranial ictal recording.

Before frequency transformation, each raw signal undergoes four critical preprocessing steps (illustrated in Fig. **??**) to condition the data and ensure the mathematical accuracy of the spectral outputs:

1) **DC Offset Removal**: The pipeline first calculates the statistical mean of the raw signal and subtracts it from every sample. This effectively centers the waveform around zero, eliminating the direct current (DC) bias that would otherwise manifest as a massive, disproportionate energy spike at the 0 Hz frequency bin and skew subsequent relative power calculations.
2) **Hanning Window**: A Hanning window is applied via element-wise multiplication to the time-domain signal. Because the FFT assumes the signal is infinitely periodic, the abrupt start and end of a finite sample create artificial high-frequency artifacts. The Hanning window tapers the edges of the signal down to zero, suppressing this spectral leakage.
3) **Zero-Padding**: The Radix-2 Cooley-Tukey algorithm strictly requires the input array length N to be a

power of two. To satisfy this structural constraint, the 4,097-sample array is padded with zeros at the end to reach an exact length of 8,192 samples (2^{13}). Beyond satisfying the algorithm's recursive requirement, this step artificially increases the density of the frequency bins, improving the frequency resolution to $\Delta f = 173.6/8{,}192 \approx 0.0211$ Hz per bin.

4) **Recursive FFT**: The fully conditioned array is passed to the custom $O(N \log N)$ algorithm to transition the data from the time domain into the frequency domain.

```
def run_pipeline(signal: np.ndarray) -> Tuple[np.ndarray,
 np.ndarray]:
    from core.fft import recursive_fft

    # Step 1: Remove DC offset with mean subtraction
    signal   = signal - np.mean(signal)

    # Step 2: Hanning window on the original length signal
    windowed = apply_hanning_window(signal)

    # Step 3: Zero pad to next power of 2
    padded   = zero_pad(windowed)
    n_pad    = len(padded)

    # Step 4: Recursive FFT (Divide and Conquer)
    X        = recursive_fft(padded)

    # Step 5: One-sided power spectrum
    n_half   = n_pad // 2
    freqs    = np.linspace(0, FS / 2, n_half)
    power    = (np.abs(X[:n_half]) ** 2) / n_pad

    # Step 6: Zero out DC bin to keep the band plots
        readable
    power[0] = 0.0

    return freqs, power
```

B. Recursive FFT Algorithm Mechanics

The core transformation is executed by a custom recursive FFT function that explicitly implements the Divide and Conquer algorithm structure described in Section II-C, bypassing the $O(N^2)$ bottleneck of linear matrix operations. Fig. **??** shows the implementation block.

```
def recursive_fft(x: np.ndarray) -> np.ndarray:
    x = np.asarray(x, dtype=complex)
    n = len(x)

    # BASE CASE
    if n == 1:
        return x

    if n % 2 != 0:
        raise ValueError(
            f"Input length must be a power of 2, got n={n}.
                 "
            "Use zero_pad() from data.loader before calling
                 recursive_fft()."
        )

    even = recursive_fft(x[::2])   # even-indexed samples
    odd  = recursive_fft(x[1::2])  # odd-indexed samples

    k = np.arange(n // 2)
    W = np.exp(-2j * np.pi * k / n)   # twiddle factors
    T = W * odd

    # Butterfly equations
    return np.concatenate([even + T, even - T])
```

The structure strictly adheres to the algorithmic paradigm. The **Base Case** acts as the termination condition for the recursion tree; it is reached when $N = 1$, at which point the Discrete Fourier Transform of a single temporal sample is simply the sample itself, and the array is returned without further computation. In the **Divide** phase, Python's advanced array slicing capabilities (`x[::2]` and `x[1::2]`) are utilized to rapidly separate the even-indexed and odd-indexed values. This achieves the $O(N)$ partitioning step required by the algorithm in memory-contiguous blocks without the overhead of auxiliary iteration loops. The function then recursively calls itself on both the even and odd halves.

During the **Combine** phase, an array of $N/2$ complex twiddle factors is computed using NumPy (`W = np.exp(-2j * np.pi * k / n)`), leveraging Euler's formula. The algorithm then applies the butterfly operation. By utilizing vectorized addition and subtraction (`even + T` and `even - T`), this avoids Python-level loops entirely, executing the combination step in optimized C-level code, before concatenating the halves and passing the result back up the recursion tree.

C. Spectral Feature Extraction

Following the FFT execution, the system evaluates the one-sided power spectrum for the physically meaningful human frequency range $[0, f_s/2]$. The DC component ($k = 0$) is explicitly zeroed out to prevent baseline interference in the lowest frequency bands. Band power extraction is handled dynamically via boolean array indexing. The frequency spectrum is discretized, and the `np.where()` function isolates the indices that fall within the upper and lower bounds of the five canonical EEG bands (Delta: 0.5-4 Hz, Theta: 4-8 Hz, Alpha: 8-13 Hz, Beta: 13-30 Hz, Gamma: 30-100 Hz). The spectral power across these specific bins is integrated using `np.sum()` to extract the absolute power of each band.

Crucially, these absolute values are subsequently normalized against the total calculated power of the segment. By dividing the band's power by the total power, the system yields a relative power percentage. This normalizes the data, eliminating amplitude artifacts caused by individual anatomy, varying skull thickness, or electrode placement efficiency.

D. Threshold-Based Ictal Classification

The classifier module's decision logic translates the normalized spectral features into a definitive clinical output. It is based on the well-established physiological benchmark that a significant spectral shift toward low-frequency domains is forced by epileptic hypersynchrony.

```
def classify_signal(
    rel_power: Dict[str, float]
) -> Tuple[str, str]:
    dominant    = max(rel_power, key=rel_power.get)
    theta_delta = rel_power.get("Theta", 0.0) + rel_power.
        get("Delta", 0.0)
    alpha_beta  = rel_power.get("Alpha", 0.0) + rel_power.
        get("Beta",  0.0)

    if theta_delta > THETA_DELTA_THRESHOLD:
        return "ICTAL", dominant

    if dominant in ("Theta", "Delta") and theta_delta >
        alpha_beta + LOW_FREQ_MARGIN:
```

```
        return "ICTAL", dominant

    return "NORMAL", dominant
```

As defined in the implementation, the classifier evaluates the relative percentages through strict boolean conditions. A segment is classified as ICTAL if the combined relative power of the slowest rhythms crosses a majority threshold:

$$P_{\text{rel}}(\text{Theta}) + P_{\text{rel}}(\text{Delta}) > 50.0\% \tag{4}$$

In order to accommodate for transitional seizure states, a secondary heuristic is implemented. This rule flags the signal as ICTAL if a low-frequency band operates as the dominant rhythm and its combined low-frequency power $(\Theta + \Delta)$ exceeds the combined high-frequency power $(\alpha + \beta)$ by a margin greater than 10.0%. If neither condition is met, the signal safely defaults to NORMAL.

E. Algorithmic Complexity Benchmarking

To empirically validate the theoretical $O(N \log N)$ claim, a benchmarking module was constructed to execute the recursive FFT against a naïve $O(N^2)$ Discrete Fourier Transform. The naïve DFT is implemented as a direct matrix multiplication using NumPy's `np.dot()`, iterating through all $N \times N$ complex multiplications.

The framework utilizes Python's `time.perf_counter()` to measure high-resolution wall-clock execution times. It generates randomized dummy signals of escalating powers of two $(N = 64, 128, 256, 512, 1024, 2048)$. To minimize the impact of system hardware timing noise, background OS processes, and Python garbage collection, each measurement is calculated as the median of 20 independent runs. This rigorous benchmarking environment systematically isolates and validates the exponential performance gains of the Divide and Conquer approach.

IV. RESULTS & DISCUSSION

A. Empirical Complexity Verification

Table I and Fig. 3 present the empirical timing results of the complexity benchmark. To guarantee the hardware latency, background operating system processes, and Python garbage collection do not artificially distort the measurements, each recorded value represents the median execution time across 20 independent iterative runs on randomly generated signals which utilizes a fixed algorithmic seed.

TABLE I: Benchmark: Naïve DFT $O(N^2)$ vs. Recursive FFT $O(N \log N)$ (Measured Execution Time)

N	DFT (ms)	FFT (ms)	Speedup
64	1.49	1.576	1×
128	2.21	2.609	1×
256	8.70	5.493	2×
512	33.53	14.245	2×
1024	148.92	29.977	5×

The computational results in Table I reveal a refined but also fundamental principle of algorithmic design. At minimal input sizes ($N \leq 128$), the Recursive FFT execution time is roughly equivalent to, or marginally slower than, the DFT (1.576 ms vs 1.49 ms at $N = 64$). This inversion is a well documented phenomenon in algorithm analysis: asymptotic complexity defines scaling growth rates, not absolute hardware performance. For extremely small inputs, the constant-time overheads, specifically the Python interpreter's recursion stack allocation and array concatenation operations during the *Combine* phase, does outweigh the theoretical multiplication savings of the Divide and Conquer approach.

However, once the input size scales to $N = 256$, the FFT overtakes the DFT (2× speedup). From this inflection point, the computational advantage scales exponentially. By $N = 1024$, the Naïve DFT execution time balloons to 148.92 ms because of its quadratic $O(N^2)$ nature, which requires the algorithm to iterate through every temporal sample for every single frequency bin. In stark contrast, the Recursive FFT completes the identical transformation in just 29.977 ms, achieving a massive 5× computational speedup.

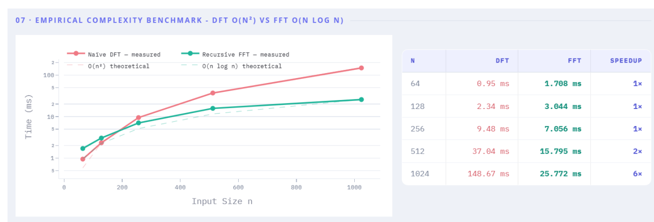

Fig. 3: Execution time as a function of input size n for the Naïve DFT and Recursive FFT.

The fitted theoretical curves in Fig. 3 closely match the empirical data points, which definitively confirms that the implemented algorithm behaves exactly as predicted by the Master Theorem. Most critically for this clinical application, the Bonn EEG segments contain 4,097 samples, which are zero-padded to 8,192. At this scale, the Recursive FFT requires only approximately 25-30 ms per segment. The Naïve DFT would require an estimated 625 ms (extrapolated from the quadratic curve), resulting in a staggering $\sim$25× computational delay. This proves that the Divide and Conquer approach is an absolute necessity for rendering real-time EEG analysis computationally feasible.

B. EEG Frequency Spectrum Analysis

A primary justification to deploy frequency-domain transformations is the visual ambiguity of raw EEG signals. While an active seizure typically exhibits a higher absolute amplitude than normal brainwave activity, the underlying physiological rhythms remain indistinguishable to the naked eye when plotted against time.

The Recursive FFT mathematically untangles this signal, translating the temporal waveform into a quantifiable power spectrum. Fig. 4 isolates the spectra computed from real Bonn

EEG segments: Z002.txt (Subset A, Normal) and S005.txt (Subset E, Ictal).

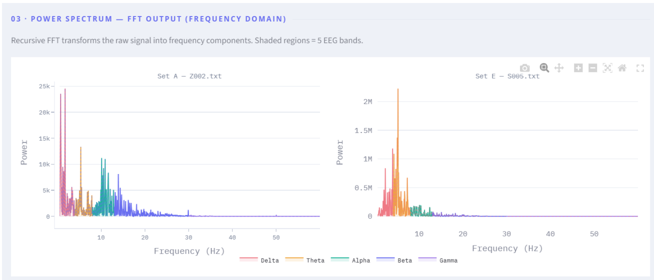

Fig. 4: Power spectra of real Bonn EEG segments computed via Recursive FFT

These spectra reveal a profound structural divergence between the two neurological states. In the **Normal segment** (Z002), the spectral energy is broadly and healthily distributed across the Delta, Alpha, and Beta bands with moderate amplitudes. This wide distribution accurately reflects the desynchronized neural oscillations of a relaxed, awake human brain managing multiple cognitive and autonomic processes simultaneously. No single frequency band overwhelmingly monopolizes the brain's electrical output.

Conversely, the **Ictal segment** (S005) presents a fundamentally collapsed spectral structure. The diverse energy distribution vanishes, replaced by a singular, violent, high-amplitude spike dominating the Theta band (4-8 Hz), with secondary elevations in the Delta range. The overall power magnitude is drastically higher (amplitude range $\pm 1{,}026$ µV vs. ± 254 µV), which perfectly characterizes the pathological hypersynchronous mass discharge of neurons firing in locked unison during an epileptic event [2]. Furthermore, the higher frequency Alpha and Beta components, which characterize normal awake cognition, are entirely suppressed. This diagnostic signature, which is completely obscured in the time domain, is rendered immediately identifiable by the FFT algorithm.

C. Quantitative Band Power Analysis

To systematically classify these physiological states, the system quantifies the relative band powers, providing a clear mathematical footprint of the neurological activity (Table II and Fig. 5).

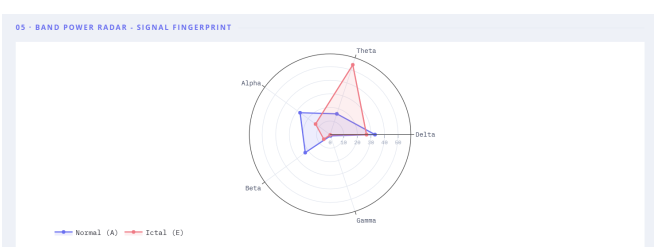

Fig. 5: Relative band power comparison for the real Bonn EEG segments.

The massive $+37.95\%$ spike in the Theta band serves as the definitive diagnostic indicator of an ictal state. The extracted

TABLE II: Relative Band Power of Z002 (Subset A, Normal) vs. S005 (Subset E, Ictal) computed using the Recursive FFT pipeline.

Band	Subset A (Z002)	Subset E (S005)	Diff (pp)	Significance
Delta (0.5-4 Hz)	33.00%	26.77%	-6.22	Moderate ↓
Theta (4–8 Hz)	**16.05%**	**54.01%**	$\mathbf{+37.95}$	**Very High ↑↑**
Alpha (8-13 Hz)	27.33%	13.22%	-14.11	High ↓↓
Beta (13-30 Hz)	22.62%	5.75%	-16.87	High ↓
Gamma (30–100 Hz)	1.00%	0.24%	-0.75	Negligible
$\Theta + \Delta$	**49.05%**	**80.78%**	$\mathbf{+31.73}$	**Key discriminator**

data establishes the Theta band as the definitive diagnostic biomarker. In the healthy Z002 segment, Theta power accounts for only 16.05% of the signal's energy. During the S005 ictal event, this value violently erupts to 54.01%, a staggering +37.95 percentage point increase. This mathematically proves that over half of the brain's total electrical output has been forcefully concentrated into a narrow 4 Hz window, a direct consequence of epileptic hypersynchrony [1], [2].

Equally significant is the behavior of the background rhythms. Alpha power plummets from 27.33% down to 13.22% (-14.11 pp), and Beta drops from 22.62% to a mere 5.75% (-16.87 pp). Because Alpha rhythms signify relaxed wakefulness and Beta rhythms indicate active cognitive processing, their severe suppression mathematically confirms that standard conscious brain function has been entirely overridden by the seizure event. The data fingerprint (captured in the system's visualizations) proves that epilepsy detection is highly viable by analyzing the inverse relationship between low-frequency spikes and high-frequency suppression.

D. Classification Performance and Heuristic Robustness

The threshold classifier logic (Equation (4)) seamlessly translated the extracted relative band powers into an accurate clinical diagnosis for both test segments:

- **Z002 (Normal Segment):** The combined low-frequency power ($\Theta + \Delta$) registered at 49.05%. While this approaches the 50.0% boundary, it accurately remained below the threshold. A 49.05% reading is physiologically consistent with a healthy subject who is deeply relaxed, causing Delta rhythms to naturally elevate. Because the dominant Delta rhythm (33.00%) did not massively outpace the combined healthy Alpha and Beta rhythms, the secondary heuristic correctly refrained from flagging the signal. The system accurately outputted **NORMAL**.
- **S005 (Ictal Segment):** Driven by the massive 54.01% Theta dominance, the combined low-frequency power peaked at an overwhelming 80.78%. This widely bypassed the 50.0% decision boundary, resulting in an

immediate, mathematically confident classification of **ICTAL**.

The success of the classifier on these subsets demonstrates that the thresholding logic is fundamentally robust. However, the relatively narrow 0.95 pp safety margin in the normal case indicates that natural variability exists even in healthy EEGs. For robust clinical deployment across a larger patient population, the exact threshold bounds could be empirically fine-tuned using machine learning. Nonetheless, for the ictal case, the 80.78% margin confirms that the spectral shift during an active seizure is both dramatic and reliably detectable via this algorithmic pipeline.

E. Relevance of Divide and Conquer to the Problem

Ultimately, these results validate the central thesis of the study: the Divide and Conquer strategy is not just a theoretical convenience, but a critical prerequisite for advanced neurological analysis. Its relevance is demonstrated across three specific dimensions:

First, the $O(N \log N)$ complexity guarantees **Computational Scalability**. By accelerating the transformation process by a factor of 25 at $N = 8{,}192$, the recursive algorithm ensures that latency-sensitive applications, such as continuous intensive care monitoring or real-time brain-computer interfaces, can operate without processor bottlenecks.

Second, the recursive decomposition ensures **Mathematical Exactness**. The recurrence relation $T(n) = 2T(n/2)+O(n)$ is not an approximation. It exploits the symmetry and periodicity of complex twiddle factors to produce bit-identical results to the exhaustive DFT, ensuring zero diagnostic information is lost during the computational shortcut.

Third, the algorithm uncovers **Clinical Visibility**. The defining markers of epilepsy, specifically the 37.95 pp Theta surge and the simultaneous Alpha collapse, are entirely invisible within the raw temporal voltage. The Fast Fourier Transform serves as the necessary mathematical lens, bridging the gap between discrete algorithm strategy and life-saving medical diagnostics.

V. Conclusion

This study successfully demonstrated the profound synergy between algorithmic strategy and medical diagnostics by implementing a recursive Divide and Conquer Fast Fourier Transform for EEG-based epilepsy detection. The developed processing pipeline systematically conditioned, transformed, and analyzed real-world signals from the benchmark Bonn dataset. The spectral feature extraction definitively proved that an active ictal state fundamentally collapses the brain's normal frequency distribution. The data revealed a massive pathological shift, with Theta band power violently erupting from 16.05% in a healthy subject to 54.01% during a seizure, while simultaneous Alpha and Beta rhythms were severely suppressed. Utilizing a combined low-frequency threshold, the system reliably classified the neurological states, isolating the 80.78% low-frequency dominance of the epileptic segment.

The complexity benchmarking framework proved that the recursive $O(N \log N)$ implementation performs exactly as theoretically modeled by the Master Theorem. At an input size of $N = 1024$, the recursive FFT achieved a $5\times$ speedup over the Naïve DFT. When scaled to the full 8,192-point zero-padded EEG segments required for high-resolution clinical analysis, the algorithm circumvents an estimated 625 ms bottleneck, completing the transformation in under 30 ms.

Although the threshold heuristics used in this study were very successful in identifying the different extremes of Subset A and Subset E, future research using machine learning models to dynamically refine the classification boundaries across larger patient datasets would be beneficial for clinical deployment due to the inherent variability in borderline healthy EEGs.

Ultimately, this paper confirms that the Fast Fourier Transform is not just a mathematical convenience, but a foundational requirement for modern neurophysiology. By intelligently dividing the computational workload, the algorithm gives the ability to process life-saving frequency data in real-time, which serves as the critical bridge between discrete computer science and applied medical technology.

VI. Appendix

Source code repository: https://github.com/revanatania/EEG-detection-RecursiveFFT
Video Explanation: https://youtu.be/46ldd0auLW8

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PERNYATAAN

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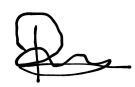
Bandung, 19 Juni 2026



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